In perfect competition we know that marginal revenue=marginal costs=price=average costs.

Since \( C(q_i) = 0 \) we know that marginal costs are zero for the entire market.
\[
C'(Q_m) = 0 = P = 240 - Q_m
\]

Therefore by simple algebra:
\[
Q_m^* = 240 = \text{The equilibrium quantity in the perfectly competitive market.}
\]

(Remember, that the perfectly competitive solution is "efficient", and therefore there is no waste, so only what will be consumed is produced.)

b. Find the following if the market is controlled by a monopolist:
   i. Quantity=120
   ii. Price=$120
   iii. Profit=$14400
   iv. Deadweight loss under a monopolist=$7200

The Monopolist solves her profit maximization problem by doing the following:
\[
\pi = P(Q_m)Q_m - C(Q_m) = (240 - Q_m)Q_m - 0 = 240Q_m - Q_m^2
\]
F.O.C.:
\[
\frac{\partial \pi}{\partial Q_m} = 0 = 240 - 2Q_m
\]
Therefore: \( 240 = 2Q_m \Rightarrow Q_m^* = 120 \)
\[
P = 240 - Q_m = 240 - 120 = $120
\]
\[
\pi = 240(120) - 120^2 = $14400
\]

Deadweight Loss:
\[
\frac{1}{2}*(120-0)(240-120) = $7200
\]

b. Find the Following if the market is controlled by two firms who make their quantity decision at the same time:
   i. Quantities supplied by the individual firms: 80 each
   ii. Price=$80
   iii. Profit=$6400 per firm
   iv. Deadweight loss under a Cournot duopoly=$3200
The two firms solve their profit maximization problem by doing the following:

\[ \pi_1 = P(Q_m)q_1 - C(q_1) = P(q_1 + q_2)q_1 - C(q_1) = (240 - q_1 - q_2)q_1 = 240q_1 - q_1^2 - q_1q_2 \]

F.O.C.:

\[ \frac{\partial \pi_1}{\partial q_1} = 0 = 240 - 2q_1 - q_2 \]

\[ q_1^* = \frac{240 - q_2}{2} = 120 - \frac{q_2}{2} \quad (1) \]

\[ \pi_2 = P(Q_m)q_1 - C(q_1) = P(q_1 + q_2)q_2 - C(q_2) = (240 - q_1 - q_2)q_2 = 240q_2 - q_2^2 - q_1q_2 \]

F.O.C.:

\[ \frac{\partial \pi_2}{\partial q_2} = 0 = 240 - 2q_2 - q_1 \]

\[ q_2^* = \frac{240 - q_1}{2} = 120 - \frac{q_1}{2} \quad (2) \]

Now we have 2 equations and 2 unknowns. By substituting firm 2’s reaction function into firm 1’s, we can solve!

\[ (2) \rightarrow (1) \Rightarrow q_1^* = 120 - \left( \frac{120 - \frac{q_2^*}{2}}{2} \right) = 120 - 60 + \frac{q_1^*}{4} \]

\[ \therefore \frac{3}{4} q_1^* = 60 \Rightarrow q_1^* = 80 \quad (3) \]

\[ (3) \rightarrow (2) \Rightarrow q_2^* = 120 - \frac{80}{2} = 80 \]

\[ q_1^* = q_2^* = 80 \]

\[ Q_m^* = q_1^* + q_2^* = 80 + 80 = 160 \]

\[ P = 240 - 160 = $80 \]

\[ \pi_1 = 240q_1 - q_1^2 - q_1q_2 = (240 * 80) - (80 * 80) - (80 * 80) = $6400 \]

\[ \pi_2 = 240q_2 - q_2^2 - q_1q_2 = (240 * 80) - (80 * 80) - (80 * 80) = $6400 \]

Deadweight Loss:

\[ (1/2)*(80-0)(240-160)=3200 \]

c. Find the Following if the market is controlled by two firms, where one firm chooses her quantity first, and the second firm follows sequentially:

i. Quantity: \( q_1=120, q_2=90 \)

ii. Price=$60

iii. Profit: Firm1=$7200, Firm2=$3600

iv. Deadweight loss under a Stackelberg duopoly=$1800
The first firm knows firm 2's strategy (Firm 2 thinks she is playing the Cournot game), and exploits this information: Firm 1 will take what he knows about firm 2, and use it directly in his profit equation.

So, rather than maximizing, and then reacting, firm 1 is now exploiting the information right at the beginning.

\[ \pi_1 = P(Q_w)q_1 - C(q_1) = P(q_1 + q_2^*)q_1 - C(q_1) = (240 - q_1 - q_2^*)q_1 \]

Where \( q_2^* = 120 - \frac{q_1}{2} \) \( (1) \)

\[ \therefore \pi_1 = 240q_1 - q_1^2 - q_1\left(120 - \frac{q_1}{2}\right) = 240q_1 - q_1^2 - 120q_1 + \frac{1}{2}q_1^2 = 240q_1 - 120q_1 - \frac{1}{2}q_1^2 \]

F.O.C.:

\[ \frac{\partial \pi_1}{\partial q_1} = 0 = 240 - 120 - q_1 \]

\[ q_1^* = 120 \] \( (2) \)

This is the key! Firm 1 has now dictated what firm 2 will do because he knew firm 2's strategy ahead of time. Firm 1 has "stolen" more of the market share, and firm 2 gets what is leftover.

\[ (2) \rightarrow (1) \Rightarrow q_2^* = 120 - \frac{120}{2} \]

\[ q_2^* = 60 \]

\[ Q_w = 120 + 60 = 180 \]

\[ P = 240 - 180 = $60 \]

\[ \pi_1 = 240q_1 - 120q_1 - \frac{1}{2}q_1^2 = (240 \times 120) - (120 \times 120) - \frac{1}{2}(120 \times 120) = $7200 \]

\[ \pi_2 = 240q_2 - q_2^2 - q_1q_2 = (240 \times 60) - (60 \times 60) - (120 \times 60) = $3600 \]

Deadweight Loss:

\[ (1/2)(60-0)(240-180) = $1800 \]

3. Explain how total surplus, consumer surplus and producer surplus are affected when monopolists are able to discriminate among consumers. (Compare and contrast surplus under perfect competition, monopoly and monopolists who discriminate.)

When changing from Monopoly to a Discriminating Monopoly: the total surplus increases, consumer’s surplus decreases, producer’s surplus increases.

When changing from Perfect Competition to a Discrimination Monopoly: the total surplus remains constant, the consumer’s surplus decreases, the producer’s surplus increases.